CATEGORIES OF CRITICAL MATHETATICS BASED REFLECTIONS ON CLIMATE CHANGE

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In this paper, we develop a theoretical framework on categories of critical reflections relevant for mathematics education. The framework is based on key concepts from critical mathematics education associated with critical citizenship. The framework adds ideas to, and combines, previous ideas and categories of critical reflections within mathematics and mathematical modelling and their applications. The framework can increase awareness of critical perspectives when addressing climate change in mathematics education research or when planning teaching contents on climate change. It can also be used as a tool for analysing critical reflections in empirical studies where climate change is the topic.

INTRODUCTION

Mathematics plays a central role in the *description*, *prediction* and *communication* of climate change (Barwell, 2013; Barwell & Suurtamm, 2011). Climate change is for instance described quantitatively based on measurements and statistical analyses of temperature, rainfall and sea level. The prediction of climate change within climate science relies on the application of advanced mathematical models while the communication of climate change includes various texts that typically make use of graphs charts and diagrams (Barwell & Suurtamm, 2011). Mathematics has a central role in shaping understandings of what climate change is and how society can respond to it. Mathematical concepts become part of the everyday language when talking about climate change (Barwell, 2013). For instance, when we talk about the climate getting warmer or colder we are in effect talking about mathematical constructs which corresponds to some measures for changes in global average temperature. These are computed through mathematical models where choices need to be made to reformulate the topic as a mathematical problem. Uncertainty in the knowledge base for climate change may be hidden in such reformulations and mathematical concepts may contribute in narrowing what is thought of as relevant for discussing climate change. The topic of climate change thus illustrates what Skovsmose (1992) refers to as the formatting power of mathematics (Barwell, 2013).

Skovsmose argues that mathematical literacy should include abilities that move beyond calculations and formal techniques and uses the notion of *mathemacy* to cover such abilities as well. Mathemacy can be defined as “a capacity of making responses and as reading the world as being open to change” (Skovsmose, 2012, p. 94). He defines mathemacy to consist of three different competences: *mathematical, technological and reflective knowing*. Mathematical knowing refers to mathematical skills and competences, technological knowing to being capable of applying mathematics and reflective knowing to recognising the formatting power of mathematics (Skovsmose, 1994).

Facilitating reflective knowing among students, associated with climate change, could include both developing mathematics based arguments in a climate change context or reflecting on mathematics based arguments produced by others, for instance by the Intergovernmental Panel on Climate Change (IPCC) or by climate sceptics in the media. A recent scoping survey on what teachers who work with climate change in mathematics classrooms do, and why, suggests that there are several ideas on how students can work with climate data to develop mathematics based arguments and critically reflect on these (Steffensen, Hansen, & Hauge, 2016). Barwell (2013) suggests that students and teachers can use data as a way to examine political issues, and look at the role mathematics can play in both creating the climate change (e.g. by facilitation of the technology), but also constructing our understanding of climate change. There are also ideas on how to critically reflect on experts’ computations (Hansen, 2012; Hauge et al., 2015).

Working with various aspects of climate change in mathematics classrooms can be valuable for the students’ present and future critical citizenship. The notion of critical citizenship can include citizen collaboration, concerns for social justice and motivation to change society (Johnson & Morris, 2010). In mathematics education this can be related to students’ knowledge and understanding of mathematics as means for self-empowerment to (re-)organize interpretations of social institutions and traditions, and for taking justified stands in social and political reforms (Skovsmose, 1994). We consider student engagement in important issues on which citizens disagree as crucial for building capacities for critical citizenship, where students for instance explore or develop mathematics based arguments. Critical mathematics education, seen for instance in the works of D’Ambrosio (2003, 2007, 2010), has further argued that being a critical citizen also requires mathematical literacy to be able to participate actively and critically in social discussions where mathematics is used.

The call for preparing students for critical citizenship matches ideas from post-normal science. This is a philosophy of science which argues that in situations where facts are uncertain, values in dispute, stakes high and decisions urgent, there may be no well-defined scientific solution to the societal problem (Funtowicz & Ravetz, 2003). Science-related policy processes, such as climate change, should also involve those who in fact have to deal with possible impacts. This idea implies that it is beneficial that non-experts are able to critically reflect on mathematics based arguments and their limitations. Taken together, there is a need to prepare students for critical citizenship, argued by both critical mathematics education and the philosophy of post-normal science. This means greater attention is needed to what such critical reflections might be.

The aim of this paper is to develop a framework of categories of critical thinking that complex societal issues may trigger when brought into mathematics classrooms. We use climate change as a case because the issue is complex, controversial and of global concern, where argumentation in the public space is often mathematics based. The involved mathematics may be too advanced for non-experts to understand, but our stance is that in a critical citizenship perspective, non-experts may have valuable capabilities in raising important questions on the assumptions embedded in the mathematics and on the role of mathematics in defining and solving the societal problem.

We build on ideas on critical reflection from post-normal science and from critical mathematics education on mathematics, mathematical modelling and statistics. In the following, we introduce some background information on literature on critical thinking related to mathematics before we introduce the framework.

IDEAS ON CRITICAL REFLECTIONS IN MATHEMATICS EDUCATION

Skovsmose (1992) developed a framework of six reflection steps, or six groups of questions, to relate types of questions to the three types of mathematical knowing. The questions listed here are examples related to each step: (1) Have we used the algorithm in the right way? (2) Have we used the right algorithm? (3) Can we rely on the result from this algorithm? (4) Could we do without formal calculations? (5) How does the actual use of an algorithm (appropriate or not) affect a specific context? (6) Could we have performed the evaluation in other way? The first two group of questions Skovsmose connected to mathematical knowing (see above) and are often answered in either a right or wrong way. Though this kind of question can suggest a true-false ideology, they are considered important for students' development of reflective knowing. The questions in the following two steps are associated with technological knowing and deal with whether an appropriate algorithm is used when solving a specific problem. These concern the relationship between tool and task, and can involve reflections on reliability. The last two groups of question focus on whether the use of a certain algorithm may shape the understanding of a problem differently than another (Skovsmose, 1992).

There are some empirical studies where the critical reflection steps are applied as analytical tools to categorize students' critical reflections in mathematics classrooms. Hauge and co-authors (Hauge et al., 2015) apply these steps when analysing preservice teachers' reflections on a graph produced by IPCC. In order to do so, adjustments were made to the group of questions. The first two groups were changed to concern the students' reflections on whether they understood the graph and underlying algorithms and critical reflections on how others, climate scientists, had applied mathematics when producing the graph. Also differentiating between the third and the fifth step has been shown to be challenging (Hauge, 2016; Hauge et al., 2015). The third step is on reliability of the approach while the fifth is on how it affects the context of the problem. These may be more or less inseparable because the context may trigger questions on the reliability of computations. For example, the attitude on how the global society should respond to global warming depends on the perception of reliability of computations.

Within statistics education there has been an emphasis on the need to understand and critically evaluate statistics used in the media and public space. This segment of the literature offers a range of arguments on why this is important, through an epistemological stance where democratic aspects of citizenship are promoted and linked to the use and misuse of statistics and statistical concepts (see for instance Gal, 2002; Mooney, & Jansem, 2011). In addition, examples from the media has been presented to offer ideas on how teachers can work with students to get experience in reading statistics in a critical way (see for instance Watson, 2004). Common for much of this literature is that it also points to specific statistical concepts and ideas and how these can mislead a reader. Although critical thinking is highlighted, there is little attention to characteristics of critical thinking similar to that of Skovsmose (1992).

Blomhøj and Jensen (2003) describe the modelling process as six sub-processes, of which each can be associated with critical reflections:

a) Formulation of a task (more or less explicit) that guides you to identify the characteristics of the perceived reality that is to be modelled.
b) Selection of the relevant objects, relations etc. from the resulting domain of inquiry, and idealization of these in order to make possible a mathematical representation.
c) Translation of these objects to mathematics
d) Use of mathematical methods to achieve mathematical results
e) Interpretation of the results regarding the initial domain of inquiry
f) Evaluation of the validity of the model by comparison with observed or predicted data, or with theoretically based knowledge.

We expect critical reflections associated with one specific sub-process to be of a different nature than another. Particularly the processes e) and f) will be in accordance with critical reflections on the formatting power of mathematics described by Skovsmose (1992).

Another framework we find relevant for our paper is Barbosa's (2006) perspectives on educational aims of working with mathematical modelling in classrooms, building on Julie (2002). Barbosa denoted these by the metaphors 1) *modelling as vehicle*, 2) *modelling as content* and 3) *modelling as critique*. These perspectives were transformed to fit the topic of climate change in a scoping survey where teachers who work with climate change in mathematics classrooms were asked what they and their students do and why (Steffensen, Hansen, & Hauge, 2015). The term *modelling* was changed to *climate change* in the educational aims. The aim within *climate change as vehicle* was to learn mathematical ideas and concepts, the aim within *climate change as content* was to learn about the topic of climate change in itself, while the aim of *climate change as critique* was to facilitate critical reflections on mathematics based discussions about climate change. We find the framework based on Barbosa relevant because each educational aim demand different categories of critical reflections, which we address in the next section.

THE FRAMEWORK

Our framework was initiated by the educational aims suggested by Steffensen, Hansen and Hauge 2015, illustrated in Figure 1. Since we, in the present paper, are particularly interested in critical reflections, we decided to investigate what critical reflections could be associated with each of the aims in Figure 1. We consider also the overlaps interesting, coinciding with Barbosas (2006, 2009) and Hansen and Hana's (2012) findings. Then we have tried to combine this with Skovsmose's (1992) six steps, or groups of questions, and critical reflections associated with statistics and mathematical modelling, as we consider all relevant for critical reflections on climate change. The resulting framework (Table 1) is thus a meta-level framework compared to that of the scoping survey, categorizing different types of critical reflections on climate change. It consists of three main

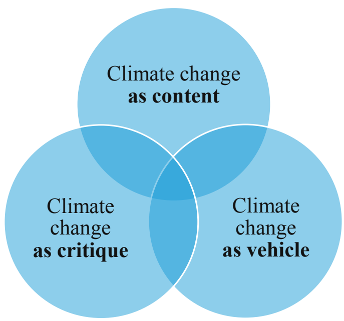

Figure 1: Visualization of different educational perspectives on climate change

categories with four overlapping categories. We have developed question groups within each category, borrowing the concept from Skovsmose's (1992) related to steps of critical reflections.

Critical reflections on *climate change as vehicle*

This category of critical reflections denotes those that are connected to the mathematics itself. Teachers may introduce the topic of climate change to learn and exercise mathematical concepts. In this case, it is valuable to reflect critically on the mathematical computations through the following question groups:

i. Are our calculations right?
ii. Are their calculations right?
iii. Was the right algorithm used?
iv. Did they use the right algorithm?
v. Have we understood the mathematical presentation?

The first and the third are the same sorts of critical reflections as Skovsmose's (1992) step i and iii. An example connected to climate change could be when students work with climate data (temperature, CO^2 emissions etc.) to learn average, spread, graphs, regression etc. Critical reflections within these group of questions, and crucial for this purpose, would be on whether the computation was correct, or whether the correct algorithm was used. Question ii and iv are similar to i and iii, except that the students consider other people's computations, for instance found in newspapers. Reflection question v concerns presentation of mathematics based arguments, as understanding a table or a graph. This is in line with what Hauge et al. (2015) found necessary in transforming Skovsmose's question i to become relevant for reflections on an IPCC graph.

Critical reflections on *climate change as critique*

The idea of this category is that working with climate change in mathematics classrooms can stimulate critical reflections on the reliability of a certain mathematical approach.

i. Is the mathematical approach reliable?
ii. Are the data or other input used in the computation of sufficient quality?
iii. Are there other ways of posing the mathematics based problem?
iv. Is the mathematics reliably presented?
v. V. Could the problem be solved without formal mathematics?

The first and last questions are the same as Skovsmose's (1992) steps three and four. We have included a question (ii) on data and other input because this is crucial for the reliability of statistical approaches and mathematical modelling. We refer to reflections on whether the mathematical problem could have been posed in another way as question iii. Questions ii and iii are in line with reflections on Blomhøj and Jensen's (2003) sub-processes b, c and d in mathematical modelling (see above). Question iv is often posed in literature on statistical literacy (i.e. Watson 1997; 2004; Gal, 2002).

Critical reflections on *climate change as content*

This area covers critical reflections which correspond to what Skovsmose's (1992) calls the formatting power of mathematics.

i. What is the role of mathematics in climate change?
ii. What are the consequences of a certain mathematics based argument for society?
iii. How can we respond to uncertainties in mathematics based arguments?
iv. What consequences are there for society when mathematics based arguments involve uncertainty?
v. How are controversies possible when arguments are mathematics based?
vi. Could we have reflected on this in another way?

The first and last questions are taken from Skovsmose's (1992) step v and vi, except that his step v, our question i, is articulated as being about climate change. The difference between question i and question ii is that question i is a more general question than ii, which refers to a certain computation of some kind. Question iii relates to reflections on how society should act on uncertainty. Because the issue of climate change is post-normal (complex, uncertain and conflicting interests), there are strong controversies. Mathematics based arguments have shown to support a range of claims, some conflicting.

Critical reflections on *climate change as vehicle/critique*

As described earlier, Skovsmose's steps are shown to be challenging to apply on recorded classroom discussions because critical reflections may seem to be in between steps or may be related to more than one step (Hauge, 2016). To address this challenge, we explore the areas which overlap the circles in Figure 1. We start with the area which covers critical reflections on climate change as vehicle and climate change as critique. We denote this overlap as critical reflections related to both the mathematics in itself and its relevance at the same time.

i. How do my choices related to data, input or defining the problem affect the results?
ii. How do their choices related to data, input or defining the problem affect the results?
iii. How does uncertainty affect the reliability of the measurement or result?

The first question is related to Skovsmose's (1992) step v, but it doesn't go all the way to reflecting on the consequences for understanding the problem, or the consequences for society. Question ii is similar to question i, except that it denotes someone else' computations. We have included the last question to differentiate between choices where the uncertainty can be discovered from exploring different choices, and actual knowledge of implied uncertainties.

The uncertainty from how a problem is posed, and its consequences, is a key issue in post-normal science, see for instance Walker et al. (2003).

Classroom activities that may foster critical reflections in this area include computations or modelling of some kind, where choices need to be made. An example could be computing the average temperature in your home town.

Critical reflections on *climate change as critique/content*

This heading we regard as involving critical aspects of mathematics based arguments in climate change that affects how climate change is understood. Such reflections concern the relevance and validity of mathematics in climate change

i. Do choices in the data, calculations or mathematical modelling affect how climate change can be understood?
ii. Would other ways of posing the problem mathematically affect how climate change can be understood?
iii. Would other ways of presenting mathematical information affect how climate change can be understood?

An example, taken from a classroom discussion on the IPCC graph on predictions of temperature change (Hauge, 2016; Hauge et al., 2015), is when one of the students comments that the idea of global warming might have been more convincing with more data back in time. Implicitly he argues that the high temperatures today would look more exceptional in a longer historical range. The student's comment could fit all questions. It could fit i as the student's topic may refer to the choice in selecting the data range, ii in that he may regard the data range as part in the posed problem, and iii in that he may refer to how mathematical information is presented in the graph. All three interpretations involve critical reflections on how choices in mathematics can affect how exceptional today's global warming is perceived.

This category of critical reflections resembles the category of critical reflections on climate change as content, but the latter reflections are rather at a meta-level of the former. For instance, the latter would be the case if the student reflected on the consequences of his suggestion for society or for decision-making.

Critical reflections on *climate change as content/vehicle*

These questions denote critical reflections in relation to learning about climate change when doing or reading mathematics.

i. How does developing my own mathematics based argument influence my understanding of climate change?
ii. How does interpreting tables, graphs or other mathematics based arguments influence my understanding of climate change?

In this case, students can collect data, work with emission time series, interpret tables or graphs, and critically reflect on what the results say about climate change. Or the students can interpret graphs etc. presented in reports or in the media.

DISCUSSION

In this paper we have developed a framework for distinguishing between various characteristics of critical mathematical reflections associated with climate change. We drew on ideas from Barbosa, Skovsmose, Blomhøj and post-normal science, which resulted in the categories of Table 1. The framework consists of three main areas, with four overlapping areas. We have not discussed the area which overlaps all three: critical reflections on *climate change as vehicle/critique/content,* but we think it is possible to imagine critical reflections sufficiently composite as to match all three criteria.

The framework adds new perspectives to categories of critical reflections, which we think are useful for understanding capacities on critical thinking in classrooms when climate change is the topic. This means that it can be used as a thinking tool both for raising awareness in research within critical mathematics education and awareness for what the teacher wishes to achieve when developing classroom activities related to climate change. In addition, the framework can be used as an analytical tool to characterize dialogues in mathematics classrooms when climate change is the topic.

The framework may be useful for a range of other topics besides climate change, but this lies outside the scope of this paper. Of course, to develop a rather detailed framework of critical reflections may be challenging as details may increase the rigidity of the categories. Also, the categories may be overlapping and confusing to apply. The framework is at its initial stage and needs careful attention in able to be developed further and examined in relation to concepts and empirical data.

REFERENCES

Barbosa, J. (2006). Mathematical modelling in classroom: a socio-critical and discursive perspective. *The International Journal on Mathematics Education, 38*(3), 293-301. doi:10.1007/BF02652812

Barbosa, J. (2009). Mathemathical modelling, the socio-critical perspective and the reflexive discussions. *Mathematics applications and modelling in the teaching and learning of mathematics;* Proceedings of TSG21 at ICME11 (pp. 133-143).

Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, *15*(1), 1-16. doi:10.1080/14794802.2012.756633

Barwell, R., & Suurtamm, C. (2011). Climate change and mathematics education: Making the invisible visible *Proceedings of the 7th Congress of the European Society for Research in Mathematics Education* (pp. 1409-1419): European Society for Research in Mathematics Rzeszów, Poland.

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching mathematics and its applications,* 22(3), 123-139.

Funtowicz, S., & Ravetz, J. (2003). Post-normal science. *International Society for Ecological Economics* (ed.).

Gal, I. (2002). Adults' Statistical Literacy: Meanings, Components, Responsibilities. *International Statistical Review, 70*(1), 1-25.

Hansen, R. (2009). Modellering og kritisk demokratisk kompetanse. *Tangenten*, 4/2009, Caspar Forlag.

Hansen, R., & Hana, G. M. (2012). But it is not possible to do this until… In G. H. Gunnarsdóttir, F. Hreinsdóttir, G. Pálsdóttir, M. Hannula, M. Hannula-Sormunen, E. Jablonka, U. T. V. Jankvist, A. Ryve, P. Valero, & K. Wæge (Eds.), *Proceedings of Norma 11: The Sixth Nordic Conference on Mathematics Education in Reykjavìk,* May 11.-14. 2011: University of Iceland Press.

Hauge, K. H. (2016). Usikkerhet i temperaturprognoser. In T. E. Rangnes & H. Alrø (Eds.), *Matematikklæring for fremtiden:* Caspar forlag.

Hauge, K. H., Sørngård, M. A., Vethe, T. I., Bringeland, T. A., Hagen, A. A., & Sumstad, M. S. (2015). Critical reflections on temperature change. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education (CERME9, 4-8 February 2015)* (pp. 1577-1583). Charles University in Prague, Faculty of Education and ERME.

Julie, C. (2002). Making relevance relevant in mathematics teacher education. In C. E. Kourouniotis, I. Vakalis, D. Hughes, D. Hallett, & Quinney (Eds.), *Proceedings of the 2nd International Conference on the Teaching of Mathematics* Wiley.

Skovsmose, O. (1992). Democratic Competence and Reflective Knowing in Mathematics. *For the Learning of Mathematics, 12*(2), 2-11.

Skovsmose, O. (2012). *An invitation to critical mathematics* education: Springer Science & Business Media.

Steffensen, L., Hansen, R., & Hauge, K.H. (2015). Climate Change in Mathematics Classroom. Paper at *ICME-13*.

Walker, W. E., Harremoës, P., Rotmans, J., Van Der Sluijs, J. P., Van Asselt, M. B. A., Janssen, P., & Krayer Von Krauss, M. P. (2003). Defining Uncertainty: A Conceptual Basis for Uncertainty Management in Model-Based Decision Support. *Integrated Assessment, 4*(1), 5-17. doi:10.1076/iaij.4.1.5.16466

Watson, J. M. (1997). Assessing statistical thinking using the media. In I. Gal & J. Garfield (Eds.), *The Assessment Challenge in Statistics Education* (pp. 107-121). Amsterdam: International Statistical Institute/IOS Press.

Watson, J. M. (2004). Quantitative literacy in the media: An arena for problem solving. *Australian Mathematics Teacher, The, 60*(1), 34-40.

Table 1. Three main categories, and three overlapping categories, of critical reflections in relation to mathematics and climate change (seen in the headings). Each category is associated with an educational aim (second row). Groups of questions, exemplifying critical reflections, are suggested within each category.

VEHICLE	**VEHICLE/ CRITIQUE**	**CRITIQUE**	**CRITIQUE/ CONTENT**	**CONTENT**	**CONTENT/ VEHICLE**
Mathematics	Mathematics and relevance	Relevance of mathematics	Relevance of mathematics and climate change	Climate change	Mathematics and climate change
i. How does mathematics influence our understanding of climate change? ii. What are the consequences of a certain mathematics based argument for society? iii. How can we respond to uncertainties in mathematics based arguments? iv. What consequences are there for society when mathematics based arguments involve uncertainty? v. How are controversies possible when arguments are mathematics based? vi. Could we have reflected on this in another way?	i. How do my choices related to data, input or defining the problem affect the results? ii. How do their choices related to data, input or defining the problem affect the results? iii. How does uncertainty affect the reliability of the measurement or result?	i. Is the mathematical approach reliable? ii. Are the data or other input used in the computation of sufficient quality? iii. Are there other ways of posing the mathematics based problem? iv. Is the mathematics reliably presented? v. Could the problem be solved without formal mathematics?	i. Do choices in the data, calculations or mathematical modeling affect how climate change can be understood? ii. Would other ways of posing the problem mathematically affect how climate change can be understood? iii. Would other ways of presenting mathematical information affect how climate change can be understood?	i. What is the role of mathematics in climate change? ii. What are the consequences of a certain mathematics based argument for society? iii. How can we respond to uncertainties in mathematics based arguments? iv. What consequences are there for society when mathematics based arguments involve uncertainty? v. How are controversies possible when arguments are mathematics based? vi. Could we have reflected on this in another way?	i. How does developing my own mathematics based argument influence my understanding of climate change? ii. How does interpreting tables, graphs or other mathematics based arguments influence my understanding of climate change?